# Appendix C RGBM Implementation[ψ]

## Iterative RGBM Implementation

As detailed, the full RGBM employs a phase volume constraint across the total dive profile. The gel parameterization is replaced by flexible seed skins with appropriate EOS, permeable to gas diffusion at all pressures and temperatures. Gas diffusion across the bubble interface, and the bubble is subject to Boyle expansion-contraction.he phase volume constraint equation is rewritten in terms of a phase function, $\phi'$, varying in time,

$$\int_0^\tau \frac{\partial \phi}{\partial t} dt \ \leq \ \Phi$$

with, as before,

$$\dot{\phi} = \frac{\partial \phi}{\partial t}$$

for $\phi$ the separated phase, and $\tau$ some (long) cutofftime. More particularly, for $\Pi$ the total gas tension,

$$\dot{\phi} = \left[\frac{\partial V}{\partial t}\right]_{diffusion} + \left[\frac{\partial V}{\partial t}\right]_{Boyle} + \left[\frac{\partial V}{\partial t}\right]_{excitation}$$

for,

$$\left[\frac{\partial V}{\partial t}\right]_{diffusion} = 4\pi DS \int_\epsilon^\infty nr \left(\Pi - P - \frac{2\gamma}{r}\right) dr$$

$$\left[\frac{\partial V}{\partial t}\right]_{Boyle} = \int_\epsilon^\infty n \left(\frac{T}{P}\frac{\partial}{\partial t}\frac{PV}{T}\right) dr$$

$$\left[\frac{\partial V}{\partial t}\right]_{excitation} = \frac{\partial}{\partial t}\left(4\pi \int_\epsilon^\infty nr^2 dr\right)$$

with all quantities as denoted previously, and the bubble number integrand normalized,

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[ψ] Extract from the book by *B.R. Wienke*, ***Reduced Gradient Bubble Model in Depth***, Best Publishing Company, ISBN 1-930536-11-9, *www.bestpub.com*

$$\int_0^{\infty} n dr = 1$$

From experiments, we employ an exponential representation in seed radii,

$$n = \beta^{-1} exp\ (-\beta r)$$

To track Boyle bubble expansion-contraction easily, a set of multipliers, $\xi$ , is tabulated in Table 1 below. For changes in pressure, we have,

$$\xi_i P_i V_i = \xi_f P_f V_f$$

as before, with i and f denoting initial and final states. Multipliers represent a 50/50 lipid-aqueous skin, following Sears, Adamson, and Epstein,

| Table 1. RGBM Boyle Multipliers | |
|---|---|
| depth (fsw) | EOS multiplier $\xi$ |
| 30 | 0.610 |
| 90 | 0.732 |
| 150 | 0.859 |
| 210 | 0.939 |
| 270 | 1.032 |
| 330 | 1.119 |
| 390 | 1.169 |
| 450 | 1.183 |
| 510 | 1.203 |

The inherent unsaturation (oxygen window), $\upsilon$, is given by by (fsw),

$$\upsilon = f_{O_2} P - 2.04(1 - f_{O_2}) - 5.47$$

with P ambient pressure, and $f_{O2}$ oxygen fraction. This window is assumed to take up inert gas under compression-decompression.

To track gas transfer across bubble boundaries, we need the mass transport coefficients, DS, for inert gases. Table 2 lists DS for the same 50/50 lipid-aqueous surface, using Frenkel, Bennett and Elliot, Harvey, Hirschfelder, and Batchelor,

| Table 2. RGBM Mass Transfer Coefficients | |
|---|---|
| Gas | DS |
| | ($\mu m^2$/sec fsw x $10^{-6}$) |
| $H_2$ | 72.5 |
| He | 18.4 |
| Ne | 10.1 |
| $N_2$ | 56.9 |
| Ar | 40.7 |
| O2 | 41.3 |

Notice that helium has a low mass transport coefficient, some 3 times smaller than nitrogen. The phase function, $\dot{\phi}$,depends on number of bubbles, n, stimulated into growth by compression- decompression, the supersaturation gradient, G, seed expansion-contraction by radial diffiusion,

$\partial r/\partial t$, Boyle expansion-contraction, PV , under pressure changes, and temperature, T, in general.

The excitation radius, $\varepsilon$, depends on the material properties, and is given for nitrogen ($\mu$m),

$$\epsilon_{N_2} = 0.007655 + 0.001654 \left[\frac{T}{P}\right]^{1/3} + 0.041602 \left[\frac{T}{P}\right]^{2/3}$$

and for helium,

$$\epsilon_{He} = 0.003114 + 0.015731 \left[\frac{T}{P}\right]^{1/3} + 0.025893 \left[\frac{T}{P}\right]^{2/3}$$

for T measured in absolute K, and P given in fsw, as before, with ranges for virial coefficients, aqueous to lipid materials, varying by factors of 0.75 to 4.86 times the values listed above. Both expression above represent fits to RGBM mixed gas data across lipid and aqueous bubble films, and are different from other phase models. Values of excitation radii, $\varepsilon$, above range from 0.01 to 0.05 $\mu$m for sea level down to 500 fsw, compared to excitation radii in other models (varying permeability and tissue bubble diffusion models) which vary in the 1 $\mu$m range. In the very large pressure limit, excitation radii (like beebees) are in the 1/1,000 $\mu$m range. Table 3 lists excitation

| Table 3. Reduced Gradient Bubble Model Excitation Radii | | | |
|---|---|---|---|
| pressure | excitation radius | pressure | excitation radius |
| P(fsw) | ε(μm) | P(fsw) | ε(μm) |
| 13 | 0.174 | 153 | 0.033 |
| 33 | 0.097 | 183 | 0.029 |
| 53 | 0.073 | 283 | 0.024 |
| 73 | 0.059 | 383 | 0.016 |
| 93 | 0.051 | 483 | 0.011 |
| 113 | 0.046 | 583 | 0.009 |

Two parameters, closing the set, are nominally (STP),

$$\Phi = 840\ \mu m^3$$

$$\beta = 0.6221\ \mu m^{-1}$$

$$2\gamma = 44.7 \left[\frac{P}{T}\right]^{1/4} + 24.3 \left[\frac{P}{T}\right]^{1/2}\ fsw\ \mu m$$